Study of Accumulation & Dynamic Decumulation (ADD) Strategy

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Executive summary

Study of an Accumulation & Dynamic Decumulation (ADD) strategy. The strategy is based on dynamic percentage which is applied on current Asset instead of the Safe Withdrawal Rule (SWR) which is applied on Asset as Date of Retirement (DoR). By construction, ADD removes the possibility of ruin. However, it creates the risk of volatile withdrawal not linked with general inflation. The risk management focus is switch to manage these volatilities.

Three strategies to reduce the volatility of withdrawal are used in the ADD strategy.

- Smoothing
    - This is a well establish method in the DB pension world. It can be naively explained as taking the moving average over 5 years. The advantage is to have a lower the volatility and assess more prudently the fund. It has the effect of delaying the impact of short term gains/loss, hopefully taking advantage of any reversion to the mean.
- Increase Cap
    - Since we take a percentage of the fund each year, exceptional returns would also imply exception increase in withdrawal. Instead, we cap the increase to a maximum versus the previous withdrawal.
- Capital Reserve
    - This technique set aside money for economic hardship. The reserve will fund the gap between 95% previous withdrawal and current withdrawal. The reserve would kick-in to fund the deficit. In economic good times, the reserve grows. The 5% gap can be seen as a bonus. Bonuses are not guaranteed. They only are often paid.

Here are the main results of this study:

1- Ruin risk is removed with ADD strategy. None should face the risk of being destitute in retirement.
2- ADD strategy is self-adjusting its withdrawal if conservative or optimistic assumption are used.
3- Sequence of withdrawal is meaningless in a fully variable withdrawal. With ADD, the risk is reduced.
4- Increase cap remove the issue of hyperinflation ruin risk. ADD mitigate hyperinflation risk in withdrawal.
5- Smoothing reduce volatility of market on withdrawal.
6- ADD reduces the retirement projection volatility.

In this paper, the ADD strategy will be shown to allow more than 40% more retirement withdrawal at retirement for less risk. During the 40 retirement years, the ADD strategy will be shown to allow for more than 60% increase on the total cash flow. This is possible by allowing some (about 1%) possibility to have to reduce future withdrawal by more than 90%.

Introduction

Decumulation strategies include: buying an annuity, a variable annuity, a deferred annuity or managed decumulation. All except the last are guaranteed by a third party. They are the most secure. However, managed decumulation advantages are that they are liquid and leave the most to the heir. This study focus on a managed decumulation strategy. One of the particularities of managed decumulation rather than the other decumulation is that it does not require a large buy at retirement. ADD use this to spread risk over multiple years.

The SWR consists of withdrawing about 4-4.5% of the fund as of Date of Retirement (DoR). The withdrawal increase according inflation after DoR. The 4-4.5% is set to be historical maximum withdrawal that would not to ruin within 30 years (1). However, mortality have substantially decrease and most retiree should assume that there's a very real chance they live longer than this after retirement. This study will thus focus on 41 retirement years. Using this retirement length, safe withdrawal rate are about 3.5%-4%.

The main disadvantage of the safe withdrawal rate strategy is ruin risk. If the future holds a worse outlook, the safe withdrawal will not be safe enough and will bring ruin. A counter argument to this would be that a rational retiree would adjust is withdrawal before such time. A better plan is to make those decisions ahead, without the need to adjust the strategy. This reduce the risk of making a bad decision during retirement. Another disadvantage is that after a decade or two, the SWR often leads to withdraw 2% of the current fund instead of the initial 4%-4.5%. SWR provides no easy path to increase withdrawal with ease of mind. The safe withdrawal rule do not prescribed when and how we should adjust course. If we start adjusting without a plan, then we would face higher ruin probability.

Instead the ADD strategy was built around the idea to adjust withdrawal along the way. The ADD strategy instead decumulates using a dynamic percentage of the fund. It smooth out the short terms variation of the fund. In wealthy time of economic growth, the withdrawal ($) and the reserve would grow. It is said that equity are correlated to the inflation (3). This should mitigate, but not remove the inflation risk. A cap is put in place to further capture some conservatism in the withdrawal. As many are risk adverse, it is better to prudently adjust withdrawal upward. At last, a reserve is created for bad economic times. In bad times, a 5% bonus withdrawal is remove from withdrawal. The reserve will kick in to keep the remaining 95% of withdrawal.

In this study we will first discuss of the main result found using this ADD strategy. These are important risk management advantage that the ADD strategy provides. At the end of that section in MR7, we compare the Safe Withdrawal Rate to the ADD strategy using a specific withdrawal rate. While the SWR provides a unique withdrawal rate, ADD strategy do not. In the Analysis section, we discuss of the Historical results using various ADD withdrawal rate. Then we discuss the Monte Carlo results using various ADD rate. We then compare what would be the ruin probability faced using SWR with the same average cash flow. The Strategy section discuss in depth of the model, the three ADD strategies and parameters used and their selection. The Data selection discusses on what Historical data that was used, what assumption for historical run was used and what assumptions was used for the Monte Carlo.

Main Result

MR1-Ruin risk is removed with ADD strategy

There's no risk of ruin with ADD withdrawal. Say you start with a million dollar and take 5% each year as withdrawal. In addition, assume that the rate of return is 0%. The first year you would withdraw 50,000$(5%*1,000,000), the next 47,500$(5%*950,000). After 10 years, you would withdraw 31,512, after 50 years 4,049$ and after 100 years 312$. The fund never goes to 0. The downside is that the withdrawal can get arbitrarily low, but not 0. SWR instead hope for no ruin risk based on historical returns. However, if rate of return are 0%, a 5% withdrawal from SWR, we would withdraw 50,000$ for 20 years. Afterward, ruin would be guaranteed.

From the point of view of the retiree, it makes no sense to risk ruin. The consequences are just too dire. Reduction strategy (return to work, reduce withdrawal) or hedging (social security benefit, pension plan) must be pursued to remove the effective ruin. The ADD strategy rather slowly adjust the withdrawal to a sustainable level given past returns.

MR2 – ADD strategy is self-adjusting

One of the truly amazing advantage is that ADD self-adjust with the current market condition. While we cannot predict the future, we know the past. Being a counter cyclical strategy, short term volatility do not affect much the withdrawal. If market worsen, a 5% Bonus will not be withdrawn. Otherwise, 95% of past withdrawal will be delivered. If in the long term economy isn't enough to support the higher ADD withdrawal rate, no increase in withdrawal will be permitted. This will result in a slow erosion in real terms while nominal withdraws stay the same. ADD would allow increase to grow only when fund are available. This act as a great protection against our greed. If we are too conservative, the ADD strategy will allow for regular increase in excess of general inflation. See Historical Analysis for more details.

MR3 – Sequence of withdrawal is meaningless in variable withdrawal

The second advantage of ADD strategy is it reduces the sequence of return risk. This is the risk of having the first few years of bad returns which cripples the fund. It's a major risk with constant withdrawal. The ADD strategy is heavily based on a variable withdrawal strategy. Thus the sequence of returns risk is reduced by using the ADD strategy. In particular using smoothing and the non-automatic increase in withdrawal creates the framework for reducing this risk.

Sequence of withdrawal is meaningless with variable withdrawal. There's a relationship between return and withdrawal. It's not as simple as a 5% returns permits a 5% withdrawal. As long as the geometric average of the fund return is in excess of 1/(1-withdrawal rate), the fund will be maintained. In other word, a 5.2632% returns permits a 5% withdrawal. Say the fund return 0% for 5 years, 10.803% (*) for 5 other years and withdraw 5% each year. If you had 1 million at the beginning, you would have 1 million ten years after. No matter where the 0% happens, the end point doesn't change.

Date	Fund	Withdraw	Interest
Yr0	1 000 000 $	50 000 $	100%
Yr1	950 000 $	47 500 $	100%
Yr2	902 500 $	45 125 $	100%
Yr3	857 375 $	42 869 $	100%
Yr4	814 506 $	40 725 $	100%
Yr5	773 781 $	38 689 $	110,803%
Yr6	814 506 $	40 725 $	110,803%
Yr7	857 375 $	42 869 $	110,803%
Yr8	902 500 $	45 125 $	110,803%
Yr9	950 000 $	47 500 $	110,803%
Yr10	1 000 000 $	50 000 $	

Date	Fund	Withdraw	Interest
Yr0	1 000 000 $	50 000 $	110,803%
Yr1	1 052 632 $	52 632 $	100,000%
Yr2	1 000 000 $	50 000 $	110,803%
Yr3	1 052 632 $	52 632 $	100,00%
Yr4	1 000 000 $	50 000 $	110,803%
Yr5	1 052 632 $	52 632 $	100,00%
Yr6	1 000 000 $	50 000 $	110,803%
Yr7	1 052 632 $	52 632 $	100,000%
Yr8	1 000 000 $	50 000 $	110,803%
Yr9	1 052 632 $	52 632 $	100,000%
Yr10	1 000 000 $	50 000 $	

(*) 10.803% = ((1/(1-5%))^10)^(1/5); Mathematically we want 1.00^5*1.10803^5*(1-5%)^10 = 1.00

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MR4 – Increase cap reduces the hyperinflation risk

The increase cap removes the issue of hyperinflation ruin risk. In Bengen(1), the Big Bang (1973-1974) and Big Dipper (1937-1941) was both period of hyperinflation. The Big Bang inflation was 22.1% and the Big Dipper inflation was 10.5%. Since SWR increase following inflation, this leads to having the model have the most stress during period of high inflation. Using a strategy that do not automatically increase with inflation has the benefit to remove the issue of going out of money, but increase the inflation risk. Many pension plan created in the 80's and recent development in the 2000's have reduced their inflation risk by going with non-automatic indexation. The ADD strategy cap increase of withdrawal as funds are available and not as inflation requires. Stocks have long been though as a partial indexation hedge. The downside being that withdrawal will not match inflation increase. On the up side, it allows for better management of the increase when rate of return allows.

In practice for the SWR, a 1966 retiree would withdraw 55,000$ out a 1.4 million portfolio at 1966. On 1981, 15 years later, the retiree would withdraw 132,000$ out of 1.4 million. Instead an ADD strategy with similar risk would withdraw more (76,000$) at 1966 and less (73,000$) at 1981. The ADD strategy would have been eroded by about 3,000$ each years due to inflation in real $ terms. ADD at the same withdrawal of 55,000$ would have allow 68,000$ on 1981 when 1.9 million of funds are available. Thus eroded by about 2,000$ each year due to inflation.

The ADD strategy would have suffered major loss due to inflation. However, SWR suggest that taking about10% withdrawal each year is safe (132,000$ out of 1.4 million). There was no guarantee for a 1981 retiree that a 50% drop in the stock market couldn't happen in 1982. A SWR follower may have been tempted to adjust his withdrawal along the way. However, without a guide, this may prove difficult. The ADD strategy provides a guide.

For reference – Historical scenario 39 using SWR withdrawal rate 3.94% at ADD withdrawal rate 6% and 4.31% at 60%/40% E/B

MR5 – ADD reduces the retirement projection volatility

The 4% rule is based off the initial fund at retirement. However, this initial fund can vary a lot if highly invested in equity. In 1995, the S&P500 made a return of 38% and inflation was 2.5%.A member who have 500,000$ of fund at the start of the 1995. The 4% rule would say that his withdrawal would be 20,000$. He would then increase it by 2.5% in 1996 and have withdrawn 20,500. If instead he had delay for 1 year his pension, then he would have 690,000$ of fund and a withdrawal of 27,600$.That's a 35% difference!

In the opposite side of the spectrum, there was a drop of 39% in returns in 2008 and 0.1% inflation. The 2008 retiree using 4% SWR would have withdrawn in 2009, 20,020$ versus a new retiree in 2009 of only 12,153$, a 40% drop.

The ADD strategy removes in large the initial year differential with the expected retirement withdrawal the prior year. In 1996, withdrawal from ADD is maximized to 8% increase than the 1995 expected withdrawal. The 2008 retiree would start with a 19,538$ and drop to 18,543$ in 2009. If he had waited 2009, he would have started with 15,714$ a 15% drop. In both situation, the ADD strategy adjust the withdrawal for the current market situation, but do not weight current market value as 100% reliable for long term withdrawal strategy.

MR6 – Smoothing reduce volatility of market on withdrawal

For simplification, thinks of smoothing used in ADD strategy as taking the average fund value of the last 5 years and adjusting it to today with a conservative return. Taking a withdrawal from this measure will be less volatile than taking a withdrawal of a constant % over the current market value. On average it makes sense to have a 5 years' worth reduction in volatility of withdrawal. If the only the smoothed fund would be available, lower volatility would be experience. Instead of a substantial drop and raise from the daily stock market, it looks at the fund through a long term approach lenses.

The historical data and Monte Carlo simulation supports this thesis. One of the tests we can do is to check the variation between one year and the next. Using 50,000 Monte Carlo simulations of 100% equity portfolio provides the insight that instead of experiencing a 18% volatility (standard deviation), the Smoothed measure and the 5 years Moving average measure both experience a 8% volatility. Similarly the Historical data also shows similar results. A 50% T-Bills & 50% US Equity also shows a reduction of volatility from 8.2% to 3.7%. A 100% T-Bills shows a modest reduction of 0.5% from 4% to 3.5%.

MR6 - Smoothing historical results

	100% Equity		50% Equity/50% T-Bills		100% T-Bills	
	classic	smooth	Classic	smooth	classic	smooth
Volatility	18,0%	8,3%	8,2%	3,7%	4,0%	3,5%
Volatility Change		46%		46%		87%

MR7 – High level analysis

The MR7 scenario looks at an initial balance of 150,000$, 20 years before retirement and a saving of 5,000$ with 3% increase each year up to retirement.

Here are some high level results that show the Safe Withdrawal Rate (SWR) over 40 years' periods. SWR guaranteed no chance of being ruined. However a single basis point extra withdrawal would cause at least one scenario where the retiree is ruined. Over the historical past at different equity to bond asset allocation (E/B), that would result in the following funds at retirement over various percentiles.

Main Result - Table 1 - Fund at retirement

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
Fund 95%	3 704 200 $	3 294 600 $	2 777 900 $	2 477 800 $	2 274 200 $	2 081 700 $
Fund 75%	2 561 400 $	2 242 400 $	1 933 200 $	1 772 700 $	1 543 900 $	1 385 900 $
Fund Avg	1 871 600 $	1 716 800 $	1 573 900 $	1 441 900 $	1 320 000 $	1 207 400 $
Fund 25%	963 400 $	950 300 $	932 700 $	925 800 $	912 400 $	886 000 $
Fund 5%	716 100 $	731 500 $	737 400 $	734 100 $	722 300 $	700 700 $

All results were rounded to the nearest 100$.

As expected the more the equity allocation, the more the fund at retirement. The 100% have more volatility than the 50% E/B. The average is below the average of the 95% and 5% percentiles. Under this investment scenario at the average, there are about 1.5 more funds with 100% E/B than a 50% E/B. At 25% percentile, the advantage erodes to 1.08 using the 100% E/B over the 50% E/B.

Main Result - Table 2 - Safe Withdrawal Rate

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
SWR Rate	3,48%	3,62%	3,74%	3,85%	3,94%	3,93%
ICF Max	170 200 $	156 100 $	141 700 $	127 800 $	114 300 $	99 300 $
ICF 95%	128 900 $	119 300 $	103 900 $	95 400 $	89 600 $	81 800 $
ICF 75%	89 100 $	81 200 $	72 300 $	68 200 $	60 800 $	54 500 $
ICF avg	65 100 $	62 100 $	58 900 $	55 500 $	52 000 $	47 400 $
ICF 25%	33 500 $	34 400 $	34 900 $	35 600 $	35 900 $	34 800 $
ICF 5%	24 900 $	26 500 $	27 600 $	28 300 $	28 500 $	27 500 $
ICF min	20 600 $	21 600 $	22 500 $	23 100 $	23 400 $	22 900 $

Using SWR over the next 40 years, 3.48% to 3.93% maximum withdrawal rates emerge from this study data. The rates alone are misleading. They would imply that the best strategy would be at the 50%/50% E/B. However if we look in dollar terms, the Initial Cash Flow (ICF) are better at the average over the 100% E/B. The conclusion at the worst case however suggests that a lower E/B of 60%/40% returns the best ICF.

The Accumulation-Dynamic Decumulation (ADD) strategy lets the retiree have a much higher return. In return, ADD introduces the risk of decreasing the withdrawal when there are economic turmoil. Let's first display the result of the ADD strategy at a dynamic 6.5% withdrawal rate. This rate comes with a high, but manageable risk. Lower risk rate are discuss afterward.

Main Result – Table 3 - Accumulation-Dynamic Decumulation

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%

ADD Rate	6,50%	6,50%	6,50%	6,50%	6,50%	6,50%
ICF max	160 500 $	157 400 $	156 200 $	147 500 $	138 300 $	128 700 $
ICF 95%	158 000 $	155 200 $	148 800 $	140 200 $	130 900 $	121 800 $
ICF 75%	129 000 $	121 400 $	116 300 $	105 700 $	93 200 $	79 900 $
ICF avg	91 000 $	89 300 $	86 500 $	82 100 $	77 100 $	71 800 $
ICF 25%	56 900 $	59 100 $	59 400 $	59 900 $	58 300 $	55 400 $
ICF 5%	40 600 $	42 300 $	43 300 $	43 800 $	43 700 $	42 900 $
ICF min	*35 300 $*	*37 300 $*	*39 200 $*	*39 700 $*	*39 600 $*	*39 000 $*

The dynamic % withdrawal of the ADD strategy is used indirectly. Often the effective withdrawal rate is more conservative.

If we compare both strategies at this stage, the volatility of ADD is much less in proportion than the SWR. ADD produces about 45% more withdrawal for only 25% more volatility. At the best historical run the ADD strategy produces less (160,500$ vs 170,200$) at the 100% E/B than the SWR. For ADD, the Average Cash Flow (ACF) in real $ terms for the 40 years following retirement can be different than ICF. Table 4 will display that. For the SWR, ACF always equals ICF by construction.

However, this does not tell how risky it is. There are two major concerns: What is the risk of having future withdrawal lower than at retirement? What is the risk of having future withdrawal lower than any previous year? The risk metric below are using nominal $ as their basis.
For the first, we need to recognize that no future retiree know what would be his withdrawal at retirement. A future retiree using the current funds can project what he expects to withdraw at retirement. The metric I Risk incorporate this by using the *percentage of run where withdrawal years* lower than 90% of the minimum between the retirement withdrawal or the average projected withdrawal (I Risk). In other word if projection prior to retirement averaged to 70,000$ and at retirement withdrawal is 71,000$, we would look for future withdrawal years below 90%*71,000$ = 63,900$. Then we looked at the average and maximum *percentage of withdrawal years within those run* (I avg & I max).
The second risk is evaluated using the *percentage of run* where a withdrawal is lower than x% of any earlier withdrawal (IIx% Risk). We also look at the average and maximum *percentage of withdrawal years within those run* (II% avg & II% max). Finally, we look at the Maximum % reduction versus any prior withdrawal years (II Max%).

Main Result - Table 4 - ADD Risk

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	6,50%	6,50%	6,50%	6,50%	6,50%	6,50%
ICF avg	91 000 $	89 300 $	86 500 $	82 100 $	77 100 $	71 800 $
ACF	108 600 $	96 900 $	85 900 $	75 400 $	65 900 $	57 200 $
I Risk	1%	0%	1%	4%	9%	13%
I avg	10%	0%	8%	10%	23%	34%
I max	10%	0%	8%	20%	58%	68%
II95% Risk	3%	3%	5%	8%	10%	14%
II95% avg	33%	35%	24%	35%	47%	43%
II95% max	35%	43%	48%	63%	75%	75%
II Max%	73%	75%	69%	62%	42%	35%

ADD is self-adjusting. 6.5% withdrawal rate is too optimistic at the 50% E/B. ACF is well below the ICF. This implies that instead of being ruined, the model slowly decreases withdrawal up to equilibrium. Now let's look at the other end of spectrum at 100% E/B. ACF double through the 41 years of withdrawal versus a 50% E/B. ACF have also been able to withstand inflation and even allows for more than ICF in the long run. The majority of the historical run (97%) didn't experience any major reduction. However, there's one case which experience a maximum reduction of 73% versus any prior withdrawal. The Average Cash Flow is greater than the initial cash flow. This means that at this level, the model was able to provide increase to the retiree in excess of the inflation.

Now let's try to get the risk under management. Instead of aiming for 6.5% withdrawal rate, let's aim for no I Risk (*percentage of run* where a withdrawal is lower than 90% of minimum between ICF and projected retirement).

Main Result - Table 5 - Metric I : FCF < ICF 90%

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	6,42%	6,53%	6,39%	6,20%	5,95%	5,75%
ICF avg	89 900 $	89 700 $	85 000 $	78 400 $	70 600 $	63 500 $
ACF Real	108 800 $	96 700 $	86 100 $	75 800 $	66 200 $	57 300 $
I Risk	0%	0%	0%	0%	0%	0%
II95% Risk	3%	4%	3%	3%	3%	3%
II95% avg	28%	27%	39%	39%	63%	58%
II95% max	35%	43%	45%	45%	70%	63%
II Max%	76%	74%	73%	71%	70%	70%

Small adjustment was made for the high E/B allocation and more adjustment for the low E/B allocation. Significant II Risk still exists at that level.. To manage the II Risk, we will set the maximum decrease in withdrawal to 85%.

Main Result - Table 6 - Metric II : FCF < 85% Any CF

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
80%*ADD rate	6,08%	6,14%	6,05%	5,88%	5,65%	5,45%
ICF avg	85 300 $	84 400 $	80 600 $	74 400 $	67 100 $	60 200 $
ACF Real	110 000 $	97 800 $	86 900 $	76 400 $	66 400 $	57 300 $
II95% Risk	1%	1%	1%	1%	1%	1%
II95% avg	23%	25%	23%	18%	25%	23%
II95% max	23%	25%	23%	18%	25%	23%
II Max%	85%	85%	85%	85%	85%	85%

Again, we see that most ICF<ACF. This implies a faster than inflation increase in future withdrawal. Also note that most of these ACF are greater than earlier table where more aggressive withdrawal rate were selected. It's another demonstration that the model self-adjust to the market. It reward in one way (better ACF) or the other (better ICF) depending on conservative or aggressive ADD rate.

If we compare SWR and ADD at this level of risk we get:

Main Result - Table 7 - SWR vs ADD with Risk II under management

ICF avg SWR	65 100 $	62 100 $	58 900 $	55 500 $	52 000 $	47 400 $
ICF avg ADD	85 300 $	84 400 $	80 600 $	74 400 $	67 100 $	60 200 $
ACF ADD	110 000 $	97 800 $	86 900 $	76 400 $	66 400 $	57 300 $

There are 30% more withdrawals if we compare SWR to ADD strategy at retirement. Over the 40 years period after retirement, the E/B asset allocations have a great impact. There's about a 14% increase if we increase the equity to bond allocation by 10%. The volatility of the fund leads to a solution of low E/B to manage risk. However, if we try to manage the withdrawal risk, we can set up strategy that instead allows for high E/B. These highs E/B over 40 years nearly double the expected cash flow if we compare a 50% E/B vs 100% E/B.

Analysis

Here, we've compiled further table to better understand the various risk of the ADD strategy.

Historical

Let's look at the various ADD withdrawal rate (WR) from 3.5% to 7% on historical result using the same scenario than MR7. The lower WR (3.5%-4%) are shown below as current (2017) market condition Monte Carlo simulations/assumptions suggest that worse outlook than historical are to be expected.

In Table 1, the benefit increase from 3.5% to 4% WR in ICF is about 14% across all E/B. However, the increase from 6.5 % to 7% WR is of only 7.5%. This is another example that ADD strategy self-adjusting. If at DoR if the fund are already stretch (at 6.5% WR), ADD conservatively doesn't allow for much more ICF at 7%.

Of particular mentions in Table 2, it is remarkable that ACFs are relatively the same across ADD withdrawal rate at the same allocation. The ICF is moving a lot, but the ACF have a high tendency of going back to the mean and provide extra withdrawal if available or lowers it if unavailable. If we compare the Average Cash Flow at various withdrawal rate, the spread is a few thousands a years versus tens of thousands in Initial Cash Flow (ICF).

Also in Table 2, the highest ACF are not at the extreme WR. They are in the 4.5%-5.5% range by a few thousands in withdrawal.

Analysis - Historical - Table 1 Average ICF

Add Rate	3,5%	4,0%	4,5%	5,0%	5,5%	6,0%	6,5%	7,0%
100%/0%	50 000 $	56 900 $	63 700 $	70 500 $	77 400 $	84 200 $	91 000 $	97 800 $
90%/10%	48 900 $	55 700 $	62 400 $	69 100 $	75 800 $	82 500 $	89 300 $	96 000 $
80%/20%	47 000 $	53 600 $	60 200 $	66 700 $	73 300 $	79 900 $	86 500 $	93 100 $
70%/30%	44 500 $	50 700 $	57 000 $	63 300 $	69 600 $	75 900 $	82 100 $	88 400 $
60%/40%	41 700 $	47 600 $	53 500 $	59 400 $	65 300 $	71 200 $	77 100 $	83 100 $
50%/50%	38 800 $	44 300 $	49 800 $	55 300 $	60 800 $	66 300 $	71 800 $	77 300 $

Analysis - Historical - Table 2 ACF

Add Rate	3,5%	4,0%	4,5%	5,0%	5,5%	6,0%	6,5%	7,0%
100%/0%	108 200 $	110 800 $	112 300 $	112 000 $	111 100 $	110 300 $	108 600 $	106 200 $
90%/10%	95 100 $	98 100 $	100 400 $	100 100 $	100 100 $	98 400 $	96 900 $	95 100 $
80%/20%	83 000 $	85 800 $	87 400 $	88 300 $	88 000 $	87 100 $	85 900 $	84 400 $
70%/30%	72 100 $	74 900 $	76 300 $	76 800 $	76 600 $	76 100 $	75 400 $	74 600 $
60%/40%	62 300 $	64 500 $	65 900 $	66 300 $	66 600 $	66 200 $	65 900 $	65 200 $
50%/50%	53 200 $	55 200 $	56 400 $	57 000 $	57 300 $	57 300 $	57 200 $	56 700 $

Analysis - Historical - Table 3 I Risk

E/B	Add Rate	3,5%-5%	5,5%	6,0%	6,5%	7,0%
100%/0%	I Risk	0%	0%	0%	1%	3%
	I avg	0%	0%	0%	10%	26%
	I max	0%	0%	0%	10%	35%
90%/10%	I Risk	0%	0%	0%	0%	6%
	I avg	0%	0%	0%	0%	22%
	I max	0%	0%	0%	0%	35%
80%/20%	I Risk	0%	0%	0%	1%	9%
	I avg	0%	0%	0%	8%	23%
	I max	0%	0%	0%	8%	40%
70%/30%	I Risk	0%	0%	0%	4%	12%
	I avg	0%	0%	0%	10%	30%
	I max	0%	0%	0%	20%	58%
60%/40%	I Risk	0%	0%	1%	9%	18%
	I avg	0%	0%	5%	23%	34%
	I max	0%	0%	5%	58%	75%
50%/50%	I Risk	0%	0%	4%	13%	17%
	I avg	0%	0%	26%	34%	45%
	I max	0%	0%	53%	68%	73%

Analysis - Historical - Table 4 II Risk

E/B	Add Rate	3,5%-5%	5,5%	6,0%	6,5%	7,0%
100%/0%	II95% Risk	0%	0%	1%	3%	8%
	II95% avg	0%	0%	15%	33%	37%
	II95% max	0%	0%	15%	35%	53%
	II Max%	95%	95%	88%	73%	55%
90%/10%	II95% Risk	0%	0%	1%	3%	15%
	II95% avg	0%	0%	3%	35%	35%
	II95% max	0%	0%	3%	43%	60%
	II Max%	95%	95%	95%	75%	47%
80%/20%	II95% Risk	0%	0%	1%	5%	14%
	II95% avg	0%	0%	18%	24%	38%
	II95% max	0%	0%	18%	48%	63%
	II Max%	95%	95%	89%	69%	42%
70%/30%	II95% Risk	0%	0%	1%	8%	14%
	II95% avg	0%	0%	33%	35%	48%
	II95% max	0%	0%	33%	63%	78%
	II Max%	95%	95%	78%	62%	35%
60%/40%	II95% Risk	0%	0%	4%	10%	19%
	II95% avg	0%	0%	48%	47%	44%
	II95% max	0%	0%	73%	75%	78%
	II Max%	95%	95%	68%	42%	29%

50%/50%	II95% Risk	0%	1%	6%	14%	19%
	II95% avg	0%	53%	47%	43%	51%
	II95% max	0%	53%	70%	75%	78%
	II Max%	95%	82%	61%	35%	25%

More risk exist with the 50% E/B allocation than the 100% E/B. This can be misleading. The Monte Carlo analysis instead revealed the opposite conclusion. One of the reasons behind this may be of lower inflation prevision. Another reason could be that historically there's some force (governmental, corporate) that results in higher than expected reversion to the mean. The Monte Carlo simulation use also no correlation between bonds and equity which may be unrealistic.

At the 7% withdrawal rate, we can see the risk of withdrawal reduction increasing rapidly. II Max% is between 25%-55%. Other strategy can be devise to handle better this extreme ADD withdrawal rate. If we would instead have a reserve that covers 85%, a 15% bonus, than the II Max% is 76% at 100% and 90% E/B. It slightly increases the II Max% to 30% at 50% E/B. However, this parameter change returns 100% I risk.

Questions that arise from these averages: Is there more risk if the ICF is low or high? Let's dissect more the 7% withdrawal rate which has some substantial risk. For this analysis, this study has sum up the risk from all 5 E/B allocation. There are no great disparities between each E/B over the risk by percentile. There is a skew over both ICF and ACF. There are no I Risk below the ICF 40% percentile. There's a single run with II risk at 90% /10% equity between 5%-25% ICF percentile. This means that if at DoR withdrawals are low, there's less risk of a drop in withdrawal. We see a disproportion of I Risk and II Risk near the top percentiles. This means that there's more risk (I and II) if ADD model predict that you are safe to withdraw a high historical ICF. Knowing that high ICF means a higher withdrawal, these retirees will be better place to reduce their withdrawal if required.

Analysis - Historical - Table 5 Risk by ICF percentile

ICF Perc.	0%-40%	40%-74%	75-94%	95%-100%
I Risk	0%	42%	38%	10%
II Risk	1%	40%	39%	13%

A similar question is: Will low ÌCF bring higher ACF? Let's look at 6.5% withawal the ACF by ICF Percentile. In the below table we see that the ACF over various ICF percentile. To answer simply the question, yes. Low ICF brings higher ACF in real $ in the future. However, it doesn't revert to the mean. In the bright side, at 100% E/B the ICF at 95% drops by about 93% in ACF, the 5% percentile almost double its withdrawal. Similarly ICF between 95%-75% drops by 3%, but ICF between 5%-25% gains 66% in future cash flow. At lower E/B, we see a less strong pull toward the mean.

Analysis - Historical - Table 6 ACF by ICF percentile

	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ICF 95%	158 000 $	155 200 $	148 800 $	140 200 $	130 900 $	121 800 $
ICF 75%	129 000 $	121 400 $	116 300 $	105 700 $	93 200 $	79 900 $
ICF avg	91 000 $	89 300 $	86 500 $	82 100 $	77 100 $	71 800 $
ICF 25%	56 900 $	59 100 $	59 400 $	59 900 $	58 300 $	55 400 $
ICF 5%	40 600 $	42 300 $	43 300 $	43 800 $	43 700 $	42 900 $
ACF at ICF 100-95%	147 300 $	146 200 $	137 700 $	122 100 $	109 700 $	96 800 $
ACF at ICF 75%-95%	125 600 $	110 300 $	96 200 $	87 300 $	85 000 $	79 300 $
ACF	108 600 $	96 900 $	85 900 $	75 400 $	65 900 $	57 200 $
ACF at ICF 5%-25%	94 500 $	86 300 $	77 100 $	67 500 $	53 400 $	47 400 $
ACF at ICF 0%-5%	78 400 $	67 200 $	56 900 $	47 800 $	42 200 $	35 400 $

A popular method of accumulation is the Target Date Fund which has a glide path approach to Equity to Bond allocation. Let’s assume that our case is age 45 at DoR – 20 years and will be 65 at DoR. Let’s see how ADD react using the following glide path:

Analysis - Historical - Table 7 Glide Path

Age	45	50	55	60	65
E/B	80%/20%	70%/30%	62%/38%	55%/45%	40%/60%

Analysis - Historical - Table 8 Target Date Fund risk

ADD Rate	5,0%	5,5%	6,0%	6,5%	7,0%
ICF avg	63 000 $	69 300 $	75 600 $	81 800 $	88 100 $
ACF avg	59 700 $	60 200 $	60 500 $	60 600 $	60 300 $
I Risk	0%	0%	6%	14%	24%
II Risk	0%	1%	8%	15%	28%

We can see that the Target Date Fund provides ICF similar to a 70/30% E/B. However, I Risk and II risk are worse than the 50%/50% E/B. These results are in-line with using higher E/B before retirement and lower than 50%/50% after retirement.

Let’s look at the impact of the Accumulation phase. Without knowing how the fund where acquired is a major concern for the ADD strategy. What’s the impact of using ADD when instead you are already near retirement? If the future retiree as access to his last balance, he could retroactively apply ADD and have better results. If not available than scenario #2 and 3 arises. Scenario #2: 500,000$ were acquired from an unknown asset mix 5 years prior to retirement, 100,000$ each year until retirement. To keep it simple, we will use a 6.5% withdrawal.

Analysis - Historical - Table 9 - Scenario #2 Fund at retirement

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
Fund 95%	2,435,900 $	2,292,100 $	2,151,900 $	2,012,400 $	1,894,800 $	1,788,700 $
Fund 75%	1,911,200 $	1,836,500 $	1,776,700 $	1,727,300 $	1,676,500 $	1,620,000 $
Fund Avg	1,612,000 $	1,574,800 $	1,538,300 $	1,502,600 $	1,467,500 $	1,433,100 $
Fund 25%	1,288,900 $	1,306,300 $	1,315,900 $	1,323,000 $	1,325,100 $	1,289,600 $
Fund 5%	808,300 $	849,300 $	890,600 $	931,900 $	972,900 $	1,013,200 $

Analysis - Historical - Table 10 - Scenario #2 Accumulation-Dynamic Decumulation

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%
ICF max	89,700 $	89,700 $	89,700 $	89,700 $	89,700 $	89,700 $
ICF 95%	80,800 $	81,600 $	82,400 $	82,400 $	82,300 $	81,800 $
ICF 75%	72,200 $	72,800 $	74,100 $	75,400 $	75,700 $	76,200 $
ICF avg	66,000 $	68,100 $	70,000 $	71,400 $	72,500 $	73,200 $
ICF 25%	60,600 $	62,600 $	65,300 $	67,200 $	68,700 $	70,000 $
ICF 5%	50,500 $	53,700 $	57,500 $	61,000 $	63,600 $	66,400 $
ICF min	36,700 $	40,600 $	44,900 $	49,400 $	54,200 $	59,400 $

Analysis - Historical - Table 11 - Scenario #2 ADD Risk

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%
ICF avg	66,000 $	68,100 $	70,000 $	71,400 $	72,500 $	73,200 $
ACF	116,500 $	106,400 $	95,600 $	86,400 $	78,300 $	70,900 $
I Risk run	0%	0%	0%	0%	0%	0%
I avg	0%	0%	0%	0%	0%	0%
I max	0%	0%	0%	0%	0%	0%
II95% Risk run	1%	0%	0%	0%	0%	0%
II95% avg	18%	0%	0%	0%	0%	0%
II95% max	18%	0%	0%	0%	0%	0%
II Max%	74%	95%	95%	95%	95%	95%

ICF is better at the lower E/B by a few thousands or tens of thousands at the lowest percentile. However, we need to be aware that the 100% E/B ACF is twice what it is at 50% E/B. Over 5 years, the greater volatility of equity outweighs its extra return. If an investor is looking to move its asset from one E/B allocation to the next he should not make any drastic moves.

Finally, Scenario #3: 500,000$ were acquired from an unknown asset mix 5 years prior to retirement, 0$ each year until retirement. To keep it simple, we will use a 6.5% withdrawal.

Analysis - Historical - Table 12 - Scenario #3 Fund at retirement

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
Fund 95%	1,421,900 $	1,321,100 $	1,219,900 $	1,125,300 $	1,064,100 $	1,008,900 $
Fund 75%	1,087,800 $	1,039,800 $	999,200 $	964,900 $	935,800 $	904,800 $
Fund Avg	894,500 $	870,100 $	846,200 $	822,800 $	799,900 $	777,500 $
Fund 25%	641,900 $	654,800 $	668,300 $	681,700 $	670,600 $	667,600 $
Fund 5%	389,300 $	419,200 $	448,100 $	475,700 $	501,900 $	527,300 $

Analysis - Historical - Table 13 - Scenario #3 Accumulation-Dynamic Decumulation

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%
ICF max	71,300 $	67,400 $	63,500 $	60,200 $	58,100 $	56,400 $
ICF 95%	57,800 $	57,200 $	56,300 $	55,200 $	54,300 $	52,900 $
ICF 75%	51,500 $	52,200 $	52,500 $	52,300 $	52,000 $	50,400 $
ICF avg	44,900 $	46,100 $	46,800 $	47,000 $	46,800 $	46,400 $
ICF 25%	39,100 $	40,800 $	41,700 $	42,800 $	43,400 $	43,500 $
ICF 5%	28,300 $	30,000 $	31,600 $	32,600 $	34,100 $	35,400 $
ICF min	15,900 $	18,300 $	20,900 $	23,700 $	26,800 $	30,100 $

Analysis - Historical - Table 14 - Scenario #3 ADD Risk

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	6.50%	6.50%	6.50%	6.50%	6.50%	6.50%
ICF avg	44,900 $	46,100 $	46,800 $	47,000 $	46,800 $	46,400 $
ACF	60,100 $	55,200 $	50,300 $	45,700 $	41,600 $	37,800 $
I Risk run	0%	0%	1%	3%	8%	12%
I avg	0%	0%	3%	10%	18%	34%
I max	0%	0%	3%	18%	38%	63%
II95% Risk run	3%	4%	4%	9%	10%	14%
II95% avg	35%	28%	42%	38%	49%	44%
II95% max	38%	45%	50%	75%	78%	75%
II Max%	70%	71%	66%	59%	42%	33%

Similarn results are found than scenario #2. ICF is better at the lower E/B by a few thousands or tens of thousands at the lowest percentile. However, we need to be aware that the 100% E/B ACF is twice what it is at 50% E/B. However, we see the disadvantage of not doing a dollar average investment. This increase the I risk II risk tremendously.

Monte Carlo

Monte Carlo results are vastly different than historical. The data, interest rate, used are substantially different than historical data. Derek Tharp has found that Monte Carlo analysis actually overstates the risk of extreme drawdowns when looking at long periods (10). With that in mind let's find out the fund available for retirement under the Monte Carlo results using 500 results.

Analysis - Monte Carlo - Table 1 - Fund at retirement

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
Fund 95%	2 585 100 $	2 279 300 $	1 994 100 $	1 718 800 $	1 501 400 $	1 312 200 $
Fund 75%	1 559 600 $	1 422 800 $	1 302 700 $	1 195 500 $	1 099 200 $	999 500 $
Fund Avg	1 310 500 $	1 217 200 $	1 128 300 $	1 043 900 $	963 800 $	888 200 $
Fund 25%	887 600 $	870 400 $	845 300 $	809 900 $	771 200 $	739 300 $
Fund 5%	574 100 $	589 700 $	598 000 $	604 300 $	596 100 $	570 500 $

These are about 25% less than the historical results. However, extreme occurrences exist. This result in very low Safe Withdrawal Rate over these 500 run shown below.

Analysis - Monte Carlo - Table 2 - Safe Withdrawal Rate

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
SWR Rate	2,35%	2,50%	2,61%	2,64%	2,66%	2,66%
ICF 95%	60 700 $	57 000 $	52 000 $	45 400 $	39 900 $	34 900 $
ICF 75%	36 700 $	35 600 $	34 000 $	31 600 $	29 200 $	26 600 $
ICF Avg	30 800 $	30 400 $	29 400 $	27 600 $	25 600 $	23 600 $
ICF 25%	20 900 $	21 800 $	22 100 $	21 400 $	20 500 $	19 700 $
ICF 5%	13 500 $	14 700 $	15 600 $	16 000 $	15 900 $	15 200 $

If we use the historical Safe Withdrawal Rate in the Monte Carlo we found that many Monte Carlo run (individual scenario) lead to Ruin. Here's a summary

Analysis - Monte Carlo - Table 3 - Historical Safe Withdrawal Rate

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
SWR Rate	3,48%	3,62%	3,74%	3,85%	3,94%	3,93%
Ruin %	5,0%	4,6%	5,0%	5,2%	5,6%	5,8%
ICF 95%	90 000 $	82 500 $	74 600 $	66 200 $	59 200 $	51 600 $
ICF 75%	54 300 $	51 500 $	48 700 $	46 000 $	43 300 $	39 300 $
ICF Avg	45 600 $	44 100 $	42 200 $	40 200 $	38 000 $	34 900 $
ICF 25%	30 900 $	31 500 $	31 600 $	31 200 $	30 400 $	29 100 $
ICF 5%	20 000 $	21 300 $	22 400 $	23 300 $	23 500 $	22 400 $

The Safe Withdrawal Rates Monte Carlo give us some confidence in retiring without having a high chance of ruin (0.2%-1.2%) using historical SWR rate. Instead if we use the ADD strategy we face no risk of ruin in the Monte Carlo analysis. With the ADD, the retiree would experience a few cases where drastic reductions in withdrawal are required in order to avoid ruin, but ruin is none the less avoided.

Below, Table 4.a displays the various impact of ADD withdrawal rate from 3.5%-7.5%. For comparison, Table 4.b shows the Safe Withdrawal Rate that match the average initial cash flow and associated risk of ruin. 500 Monte Carlo runs have been used for tables 4.a & 4.b.

One of the interesting conclusions is that ACF is always substantially greater than ICF using the Monte Carlo. Also interestingly, the SWR ruin probability is about the same as the II 95% Risk. Thus the ADD model mostly replaces the ruin risk for force the withdrawal adjustment due to the overall economy disarray.

ADD 3.5%

Analysis - Monte Carlo - Table 4.a - ADD 3,5%

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	3,50%	3,50%	3,50%	3,50%	3,50%	3,50%
ICF 95%	68 900 $	62 700 $	57 800 $	52 400 $	47 600 $	43 000 $
ICF 75%	47 400 $	45 200 $	42 600 $	39 900 $	37 200 $	34 200 $
ICF Avg	39 200 $	37 800 $	36 000 $	34 100 $	32 100 $	30 000 $
ICF 25%	28 300 $	28 100 $	27 800 $	27 400 $	26 600 $	25 200 $
ICF 5%	19 500 $	20 100 $	20 400 $	20 500 $	20 500 $	19 900 $
ACF Real	87 100 $	75 100 $	64 400 $	54 900 $	46 300 $	38 900 $
MV(65>105)	-	0,00	0,00	0,00	0,00	0,00
I Risk	1%	1%	0%	0%	0%	0%
I avg	0%	0%	0%	0%	0%	0%
I max	65%	60%	43%	0%	0%	0%
II95% Risk	1%	1%	1%	1%	0%	0%
II95% avg	1%	1%	0%	0%	0%	0%
II95% max	75%	73%	73%	73%	58%	0%
II Max%	36%	43%	50%	55%	64%	95%

Analysis - Monte Carlo - Table 4.b - SWR equivalent ADD 3,5%

SWR Rate	2,99%	3,11%	3,19%	3,27%	3,33%	3,38%
SWR ICF	39 200 $	37 800 $	36 000 $	34 100 $	32 100 $	30 000 $
SWR Ruin	2%	1%	1%	1%	1%	2%

ADD 4.0%

Analysis - Monte Carlo - Table 4.a - ADD 4,0%

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	4,00%	4,00%	4,00%	4,00%	4,00%	4,00%
ICF 95%	78 400 $	71 200 $	66 000 $	59 800 $	54 400 $	49 200 $
ICF 75%	54 200 $	51 500 $	48 700 $	45 600 $	42 400 $	39 100 $
ICF Avg	44 700 $	43 100 $	41 100 $	39 000 $	36 600 $	34 200 $
ICF 25%	32 300 $	32 100 $	31 700 $	31 300 $	30 300 $	28 800 $
ICF 5%	22 300 $	23 000 $	23 300 $	23 500 $	23 400 $	22 800 $
ACF Real	90 000 $	77 600 $	66 500 $	56 700 $	47 900 $	40 300 $
MV(65>105)	0,01	0,01	0,01	0,01	0,01	0,01
I Risk	2%	2%	1%	1%	0%	1%
I avg	1%	1%	0%	0%	0%	0%
I max	78%	75%	68%	48%	38%	38%
II95% Risk	3%	2%	2%	1%	1%	1%
II95% avg	2%	1%	1%	1%	0%	0%
II95% max	85%	78%	75%	73%	60%	68%
II Max%	32%	37%	42%	45%	56%	66%

Analysis - Monte Carlo - Table 4.b - SWR equivalent ADD 4,0%

SWR Rate	3,41%	3,54%	3,65%	3,74%	3,80%	3,85%
SWR ICF	44 700 $	43 100 $	41 100 $	39 000 $	36 600 $	34 200 $
SWR Ruin	4%	4%	4%	4%	4%	5%

ADD 4.5%

Analysis - Monte Carlo - Table 4.a - ADD 4,5%

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	4,50%	4,50%	4,50%	4,50%	4,50%	4,50%
ICF 95%	88 000 $	80 100 $	74 100 $	67 100 $	61 200 $	55 300 $
ICF 75%	60 900 $	57 900 $	54 700 $	51 300 $	47 700 $	44 000 $
ICF Avg	50 200 $	48 400 $	46 200 $	43 800 $	41 200 $	38 500 $
ICF 25%	36 200 $	36 100 $	35 700 $	35 200 $	34 100 $	32 400 $
ICF 5%	25 000 $	25 800 $	26 200 $	26 400 $	26 300 $	25 600 $
ACF Real	91 800 $	79 000 $	67 700 $	57 700 $	48 900 $	41 200 $
MV(65>105)	0,03	0,04	0,03	0,03	0,03	0,04
I Risk	4%	3%	2%	2%	2%	2%
I avg	2%	1%	1%	1%	1%	1%
I max	88%	85%	85%	78%	55%	55%
II95% Risk	5%	4%	3%	3%	3%	2%
II95% avg	3%	2%	1%	1%	1%	1%
II95% max	88%	85%	85%	78%	75%	75%
II Max%	28%	32%	35%	38%	44%	43%

Analysis - Monte Carlo - Table 4.b - SWR equivalent ADD 4,5%

SWR Rate	3,83%	3,98%	4,10%	4,20%	4,28%	4,34%
SWR ICF	50 200 $	48 400 $	46 200 $	43 800 $	41 200 $	38 500 $
SWR Ruin	6%	7%	8%	8%	10%	15%

ADD 5.0%

Analysis - Monte Carlo - Table 4.a - ADD 5,0%

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	5,00%	5,00%	5,00%	5,00%	5,00%	5,00%
ICF 95%	97 600 $	88 900 $	82 300 $	74 400 $	68 000 $	61 400 $
ICF 75%	67 600 $	64 300 $	60 800 $	56 900 $	53 000 $	48 900 $
ICF Avg	55 700 $	53 700 $	51 300 $	48 600 $	45 700 $	42 800 $
ICF 25%	40 200 $	40 100 $	39 600 $	39 000 $	37 900 $	36 000 $
ICF 5%	27 700 $	28 700 $	29 100 $	29 300 $	29 300 $	28 500 $
ACF Real	92 400 $	79 700 $	68 300 $	58 300 $	49 500 $	41 800 $
MV(65>105)	0,05	0,06	0,07	0,08	0,08	0,10
I Risk	6%	6%	5%	5%	5%	6%
I avg	3%	3%	2%	2%	2%	2%
I max	88%	88%	85%	83%	75%	70%
II95% Risk	8%	8%	7%	7%	6%	7%
II95% avg	4%	4%	3%	3%	3%	3%
II95% max	88%	88%	85%	85%	83%	80%
II Max%	24%	27%	28%	32%	33%	32%

Analysis - Monte Carlo - Table 4.b - SWR equivalent ADD 5,0%

SWR Rate	4,25%	4,42%	4,55%	4,66%	4,74%	4,82%
SWR ICF	55 700 $	53 700 $	51 300 $	48 600 $	45 700 $	42 800 $
SWR Ruin	10%	12%	13%	15%	20%	29%

ADD 5.5%

Analysis - Monte Carlo - Table 4.a - ADD 5,5%

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	5,50%	5,50%	5,50%	5,50%	5,50%	5,50%
ICF 95%	107 100 $	97 800 $	90 400 $	81 800 $	74 700 $	67 500 $
ICF 75%	74 400 $	70 700 $	66 800 $	62 600 $	58 200 $	53 800 $
ICF Avg	61 100 $	59 000 $	56 400 $	53 500 $	50 300 $	47 000 $
ICF 25%	44 100 $	44 100 $	43 600 $	42 900 $	41 700 $	39 500 $
ICF 5%	30 500 $	31 600 $	32 000 $	32 200 $	32 200 $	31 300 $
ACF Real	92 500 $	79 700 $	68 400 $	58 400 $	49 600 $	42 100 $
MV(65>105)	0,10	0,11	0,13	0,14	0,15	0,21
I Risk	9%	9%	9%	9%	10%	12%
I avg	4%	4%	4%	4%	4%	5%
I max	88%	88%	85%	85%	80%	78%
II95% Risk	14%	13%	11%	12%	11%	13%
II95% avg	6%	6%	6%	6%	6%	6%
II95% max	90%	88%	88%	85%	85%	85%
II Max%	20%	22%	24%	25%	24%	23%

Analysis - Monte Carlo - Table 4.b - SWR equivalent ADD 5,5%

SWR Rate	4,67%	4,85%	5,00%	5,13%	5,22%	5,29%
SWR ICF	61 100 $	59 000 $	56 400 $	53 500 $	50 300 $	47 000 $
SWR Ruin	15%	16%	20%	27%	34%	47%

ADD 6.0%

Analysis - Monte Carlo - Table 4.a - ADD 6,0%

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	6,00%	6,00%	6,00%	6,00%	6,00%	6,00%
ICF 95%	116 700 $	106 700 $	98 400 $	89 100 $	81 500 $	73 600 $
ICF 75%	81 100 $	77 100 $	72 800 $	68 200 $	63 500 $	58 600 $
ICF Avg	66 600 $	64 300 $	61 500 $	58 300 $	54 800 $	51 300 $
ICF 25%	48 000 $	48 100 $	47 500 $	46 800 $	45 500 $	43 100 $
ICF 5%	33 200 $	34 400 $	34 900 $	35 200 $	35 100 $	34 200 $
ACF Real	92 100 $	79 200 $	68 000 $	58 000 $	49 500 $	42 000 $
MV(65>105)	0,16	0,19	0,22	0,25	0,29	0,37
I Risk	14%	15%	16%	16%	21%	24%
I avg	7%	7%	7%	8%	9%	10%
I max	90%	88%	88%	85%	85%	80%
II95% Risk	22%	23%	22%	20%	23%	27%
II95% avg	10%	10%	10%	10%	10%	12%
II95% max	90%	88%	88%	88%	88%	85%
II Max%	16%	18%	19%	18%	18%	17%

Analysis - Monte Carlo - Table 4.b - SWR equivalent ADD 6,0%

SWR Rate	5,09%	5,29%	5,45%	5,59%	5,69%	5,78%
SWR ICF	66 600 $	64 300 $	61 500 $	58 300 $	54 800 $	51 300 $
SWR Ruin	19%	25%	30%	38%	50%	64%

ADD 6.5%

Analysis - Monte Carlo - Table 4.a - ADD 6,5%

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	6,50%	6,50%	6,50%	6,50%	6,50%	6,50%
ICF 95%	126 300 $	115 600 $	106 400 $	96 500 $	88 300 $	79 800 $
ICF 75%	87 900 $	83 500 $	78 800 $	73 800 $	68 800 $	63 400 $
ICF Avg	72 100 $	69 600 $	66 600 $	63 100 $	59 400 $	55 500 $
ICF 25%	52 000 $	52 100 $	51 400 $	50 700 $	49 300 $	46 700 $
ICF 5%	36 000 $	37 300 $	37 900 $	38 100 $	38 000 $	37 000 $
ACF Real	90 900 $	78 200 $	67 000 $	57 400 $	48 900 $	41 600 $
MV(65>105)	0,24	0,27	0,32	0,36	0,42	0,51
I Risk	20%	22%	24%	27%	32%	38%
I avg	10%	11%	12%	13%	15%	18%
I max	90%	90%	88%	88%	85%	85%
II95% Risk	30%	31%	31%	32%	35%	41%
II95% avg	14%	15%	15%	16%	17%	20%
II95% max	95%	90%	90%	90%	88%	88%
II Max%	13%	14%	14%	14%	14%	13%

Analysis - Monte Carlo - Table 4.b - SWR equivalent ADD 6,5%

SWR Rate	5,51%	5,72%	5,91%	6,05%	6,17%	6,25%
SWR ICF	72 100 $	69 600 $	66 600 $	63 100 $	59 400 $	55 500 $
SWR Ruin	28%	33%	43%	51%	64%	76%

ADD 7.0%

Analysis - Monte Carlo - Table 4.a - ADD 7,0%

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	7,00%	7,00%	7,00%	7,00%	7,00%	7,00%
ICF 95%	135 600 $	124 500 $	114 400 $	103 900 $	95 100 $	85 900 $
ICF 75%	94 600 $	89 900 $	84 800 $	79 400 $	74 100 $	68 200 $
ICF Avg	77 600 $	74 900 $	71 700 $	68 000 $	64 000 $	59 800 $
ICF 25%	55 900 $	56 100 $	55 400 $	54 600 $	53 000 $	50 300 $
ICF 5%	38 700 $	40 200 $	40 800 $	41 000 $	40 900 $	39 900 $
ACF Real	89 100 $	76 700 $	65 800 $	56 300 $	48 100 $	40 900 $
MV(65>105)	0,32	0,37	0,42	0,48	0,54	0,66
I Risk	27%	29%	34%	36%	42%	51%
I avg	14%	15%	17%	19%	21%	25%
I max	95%	90%	90%	90%	88%	88%
II95% Risk	38%	39%	42%	43%	46%	55%
II95% avg	19%	20%	20%	22%	23%	27%
II95% max	95%	93%	93%	93%	90%	90%
II Max%	11%	11%	11%	11%	10%	10%

Analysis - Monte Carlo - Table 4.b - SWR equivalent ADD 7,0%

SWR Rate	5,93%	6,16%	6,36%	6,52%	6,64%	6,74%
SWR ICF	77 600 $	74 900 $	71 700 $	68 000 $	64 000 $	59 800 $
SWR Ruin	35%	44%	52%	63%	75%	86%

ADD 7.5%

Analysis - Monte Carlo - Table 4.a - ADD 7,5%

E/B	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
ADD Rate	7,50%	7,50%	7,50%	7,50%	7,50%	7,50%
ICF 95%	144 800 $	133 400 $	122 500 $	111 300 $	101 800 $	92 000 $
ICF 75%	101 400 $	96 300 $	90 700 $	85 000 $	79 400 $	73 100 $
ICF Avg	83 000 $	80 300 $	76 800 $	72 800 $	68 500 $	64 100 $
ICF 25%	59 900 $	60 100 $	59 300 $	58 400 $	56 800 $	53 900 $
ICF 5%	41 500 $	43 000 $	43 700 $	43 900 $	43 800 $	42 700 $
ACF Real	86 900 $	74 800 $	64 200 $	55 000 $	47 000 $	40 000 $
MV(65>105)	0,43	0,48	0,54	0,59	0,70	0,80
I Risk	35%	38%	42%	49%	55%	65%
I avg	18%	20%	22%	24%	28%	33%
I max	95%	93%	90%	90%	90%	90%
II95% Risk	46%	50%	51%	55%	61%	70%
II95% avg	24%	25%	26%	28%	31%	36%
II95% max	98%	95%	93%	93%	93%	90%
II Max%	9%	9%	8%	8%	8%	7%

Analysis - Monte Carlo - Table 4.b - SWR equivalent ADD 7,5%

SWR Rate	6,34%	6,60%	6,81%	6,98%	7,11%	7,22%
SWR ICF	83 000 $	80 300 $	76 800 $	72 800 $	68 500 $	64 100 $
SWR Ruin	44%	51%	61%	74%	84%	92%

Model and strategy

This section describe the model and various strategies in detail. We also go through each of the available parameters. One of the issue with having a more complex withdrawal strategy involving many moving parts is that the system is complex. Depending on the retiree objective, many ideal parameter can emerge. This study have tested some of the possibilities, but other objectives may result in other ideal parameters setting. If one parameter is changed, the other should be review if they best fits the objective.

ADD Model in aggregate

The ADD model functions by withdrawing a certain % of the fund each year. The % of withdrawal will vary each year according to the economic situation that was previous experience. When in good times, the withdrawal rate will be lower. The extra reserve will be used when economic time worsens. Thus in bad times, the withdrawal rate will increase. While this may look counterintuitive, this is a direct following the ant insight in Aesop's fable. We need to build reserve when the sun is shining.

The funds are split into two. 75% of the account goes in the regular fund. 25% goes into the reserve fund. The withdrawal of the ADD strategy is in 2 parts, the regular and the reserve. The regular withdrawal consist of taking the minimum between the smoothed fund * prescribed withdrawal rate and an increase cap. The increase cap is based on the last year withdrawal times the maximum increase. The reserve is left to grow unless the regular withdrawal is lower than 95% of any previous year's withdrawals. The reserve covers up to the 95% mark, but not in excess of 20% withdrawal rate on the reserve. Finally in prolong great economic cycle, the reserve is slowly transfer back to the regular fund if the reserve exceeds 35% of the total fund.

Simple example of the mechanic of the ADD strategy:

Background Data:

- A few years into retirement.
- Total fund value : 419,483$
  - Regular fund value : 261,174$
    - Smoothed Regular Fund: 277,363$
  - Reserve fund value : 158,309$
- Last year withdrawal : 16,498$
  - This was not due to the increase cap.
- Greatest previous withdrawal : 19,618$

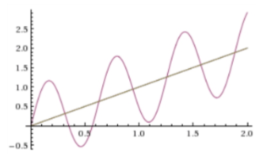
ADD withdrawal:

- Total withdrawal : 18,637$ = 17,323$ + 1,314$
  - Regular withdrawal : 17,323$ = Min (19,277 ; 17,323$)
    - Smoothed withdrawal prescribed : 24,038$ = 8.67% * 277,363$
      - *[Parameter]* ADD withdrawal rate : 6.50%
        - This drives the "prescribed withdrawal rate" to 8.67% = 6.5%/(75%). The
    - Increase Cap: 17,323$ = 16,498$ * (105% + 0%)
      - *[Parameter]* Increase Cap: 105%.

                - *[Parameter]* Increase Cap is augmented if prior year was cap: +0%.
    - Reserve withdrawal : 1,314$ = Min(18,637$ - 17,323$; 31,662$)
        - Maximum covered withdrawal : 18,637$ = 95% * 19,618$
            - *[Parameter] R*eserve protected : 95%
            - *[Parameter] R*eserve maximum withdrawal rate : 20%
                - Maximum withdrawal: 31,662$ = 20%* 158,309$
            - No adjustment required since last years and current year reserve fund =158,309/419,483 = 38% > 20%
            - In fact, the reserve exceeds the 30% threshold and will return back to the regular fund the excess 158,309 - 30%*419,483= 32,464. This can be though as a regular contribution.

Smoothing

The market value, the traditional measurement, is not in-line with its future value. The fund fluctuate similarly to an increasing sin function while competing force push up and down the market. Retirement length is often 3 to 4 decades long. Over a few years, many peaks and valleys revert to the mean. In retirement, our goal is to manage the withdrawal fluctuation not the fund worth. To better measure than the available money for a long term withdrawal strategy, we use the average MV.



When the fund is lower than the average, you would withdraw more %. When the fund is higher than you would withdraw less %. However, your fund is not as simple as a sin function. One way to apply smoothing is the linear recognition N-Year smoothing. First, estimate the fund return to be, say 10%. You then recognize 20% of the real rate of return instead of the estimate for last year, 40% 2 years prior, 60% 3 years prior, 80% 4 years prior and 100% 5 years prior. Thus the result would be somewhere near the imaginary line in the above sin function.
The main disadvantage is that smoothing will hide/delay the recognition of Gain&Loss. Thus, this method can serve you as a quick estimate, but the fund value may be vastly different. Be prepare to adjust ADD withdrawal for past event rather than current event. In other word, 2008 crash affected the smoothed average over 5 years. It helps manage the short term variations.

Simple example of the smoothing assuming a less than expected return

Data:

Year	2011	2012	2013	2014	2015	Average
Market value	500,000$	480,000$	490,000$	510,000$	530,000$	502,000$
Rate of Return	-4.0000%	2.0833%	4.0816%	3.9216%	N/A	1.7089%

- Smoothed Value with corridor at 2015 = Min(CorridorMin; Max(CorridorMax; Smoothed Value)) = 558,359$
    - CorridorMin: 530,000*0.8 = 424,000 $

        - *[Parameter]* Corridor Minimum = 80%
    - CorridorMax: 530,000*1.2 = 636,000 $
        - *[Parameter]* Corridor Maximum = 120%
    - Smoothed Value = 1/5*500,000*(-(-4%)+5.67%)+2/5*480,000*(-(2.0833%)+5.67%)+3/5*490,000*(-(4.0816)%+5.67%)+4/5*510,000*(-(3.9216)%+5.67%)+530,000 = 558,359$
        - *[Parameter]* conservative rate of return = 5.67%
        - A simple check is to compare with a 5 year moving average. Multiply the average (502,000) by 1.0567*1.0567 = 560,540$

Parameter

To smooth a fund, we need many parameters.

A. Number of Years to smooth
B. Conservative rate of return
C. Corridor : Min % of real fund; Max % of real fund

A: The foremost important is the number of year to smooth over. Using 1 year would mean that no smoothing is applied. In this study we use a 5 years average. This is in line with the economic cycle. This is also in accordance with Society of Actuary recommendation.
B: The conservative rate of return used for the smoothing average can be determined by subtracting from the arithmetic average the variance. Using such conservative rate of return over 50,000 simulations, the smoothing is unbiased versus the Market value. Similar results are also found on historical results on Canadian equity, US equity, Bonds and T-Bill.
An error in this parameter setup does not lead to a large issue. If we miss by 1%, it leads to an about 1.5% increase in ADD withdrawal. For an annual 40,000$, this leads to instead withdraw of 40,600$. That is a 50$ increase monthly. The impact is relatively small. For comparison if we increase the withdrawal rate by 1%, this leads to 7%-14% increase in withdrawal.
C: The corridor min and max should be unbiased or conservative. A bias here would lead to a consistent overvalue or undervalue of the fund. This is unwanted. Otherwise, having a sanity check on the smoothing ensure that major economic crisis or boom is reflected earlier. There's no deterministic way to determine the value of the parameter. 80%/120% was used for this study, but other reasonable corridor could be used or none at all can be used. The impact to adding it is minimal.

Capping

ADD use an increase cap to manage the inflation risk. In simple term, if we withdraw 40,000$ yesterday, we limit today withdraw to 45,000$. This mitigates the effect of financial bubble and provide a conservative strategy to manage increase. On the down side, you delay the gain of excessive return for later. To manage this, ADD increase the cap by 1% for each prior year that the cap was applied. This slowly allows the retiree to gains advantage of any sustained rise in the fund value.

A complementary strategy is to limit the projected withdraw using the same strategy. If the year before retirement, projected withdrawal was 40,000$, ADD limits the retirement withdrawal to 48,000$. The capping is greater to adjust for the 0% withdrawal prior retirement.

Simple example of the capping:

Year	1979	1980	1981	1982	1983
Smoothing Withdrawal	44,309$	47,408$	49,438$	48,428$	54,950$
Cap	N/A	44,309*105% = 46,524	46,524*106% = 49,316	49,316*107% = 52,768	48,428*105% = 50,850
Effectively withdraw	44,309$	46,524	49,316	48,428	50,850

The initial Cap is 105%. The rate of return during these years was exceptional. Without a cap, withdrawals increase by 24% in a manner of 5 years. To manage this high increase, the Cap is increase each year it was previously cap. This increase in cap insures that conservatively increase your withdrawal according to the market condition. It also have the added benefit of removing the decision of increasing your withdrawal. One of the major flaw of the classic 4% rule is that it may become too conservative (withdrawing 1%) after 10-20 years. The strategy does not have the flexibility to decide how to increase it. If you do it too often, then you are at risk of always increasing it in good time, without any buffer for adverse deviation.

Parameter

To cap a withdrawal, we need many parameters.

A. Capping percentage prior retirement
    - This is a major improvement versus the classic 4% rule. In 1995, the S&P500 made a return of 38% and inflation was 2.5%.A member who have 500,000$ of fund at the start of the 1995. The 4% rule would say that his withdrawal would be 20,000$. He would then increase it by 2.5% in 1996 and have 20,500. If instead he had delay for 1 year his pension, then he would have 690,000$ of fund and a withdrawal of 27,600$.That's a 35% difference!
    - A 115% cap is used. In theory we need to cover the increase we want post retirement and withdrawal. A more conservative lower the initial withdrawal and increase the room for automatic increase in the first few years. The 115% was determined to reduce I risk.

B. The capping percentage to apply at retirement
    - We use 8% for this study. In order to smooth out the change from the cap prior retirement to after retirement, a middle ground is found at 8% for the year at retirement. This provides an extra conservative cushion for the following years after retirement.

C. The capping percentage to apply on after retirement
    - A 106% increase is appropriate. At 7% ADD withdrawal rate the average withdrawal over 40 years adjusted for inflation is maximized and the drop in withdrawal is the lowest. Over the historical data, cap between 105% and 110% produces similar results in those metrics. Impact are experience when cap is 115% or more. This drop the II Risk Max to 55% instead of 78%. The average cash flow is somewhat reduce by a few % when looking at the cap below 105%.

D. A Sub-parameter can be to allow for Capping % to increase if earlier year was also cap.
    - This removes the issue of ending up with withdrawing 1% of the fund in the later year if economic growth was fantastic.

- For this model, we use this parameter using 1% increase per year where cap was already use. If we disable this parameter, the ACF are reduce substantially. The correct range seems to be between 0.5% and 1.5%. The lower the rate, the less the average cash flow after retirement. The higher the rate, the greater the I risk.

Reserve

Banks and insurance company alike have capital reserve. This same principle can be applied to a personal fund without any outside force. This technique is similar to the bucket approach. Say that you have 1 million at retirement. You take 250k and put it in reserve. You then withdraw 5.33% of 750k instead, that is 40k= 750k*5.33%. If the fund earns more than 5.33%, the reserve builds for future year. If it earns less, you use the reserve to reduce the impact of the loss on your pension. Notice that a 5.33% withdrawal on 75% of the fund really implies a 4.33% withdrawal on the total fund. For clarity, ADD withdrawal rate will means the withdrawal rate on the non-reserve account and the effective withdrawal rate will means the withdrawal rate on the whole account.

While we could cover 100% of the past withdrawal doing a floor approach, this strategy let the withdrawal go down if market fails. This way, we capture the most of the benefit of having a floating withdrawal rate, while managing the negative consequence.

Faced with the latest great recession, you would have been able to pull from these reserves during these harsh times. In the end, it's an accounting trick. The retiree do not create any new money. It applies a "filter" over the portfolio. It helps better manage the volatility and unpredictability of the fund.

Simple example

25% reserve covering 95% drop using an ADD withdrawal rate of 8%:

Year	2011	2012	2013	2014	2015
MV BoY	500,000$	286,700$	355,660$	350,162$	328,096$
MV NR BoY	375,000$	210,450$	251,698$	263,981$	247,720$
MV R BoY	125,000$	76,250$	83,962$	86,181$	80,376$
Withdrawal BoY	30,000$	28,500$	28,500$	28,500$	28,500$
Withdrawal NR BoY	30,000$	16,836$	20,136$	21,118$	19,818$
Withdrawal R BoY	0$	11,664$	8,364$	7,382$	8,682$
MV EoY	470,000$	258,200$	307,160$	321,662$	299,596$
MV NR EoY	345,000$	193,614$	231,562$	242,863$	227,902$
MV R EoY	125,000$	64,586$	75,598$	78,800$	71,693$
Rate of Return	-39 %	30%	14%	2%	N/A

Market Value: MV; Reserve: R; Non Reserve: NR; BoY: Begin of Year; EoY: End of Year

As expected in the financial crisis, the fund substantially shrinks during the 4 years. However, we are able to maintain a 95% of the initial withdrawal with relative ease. We can see that the fund value increase substantially after the fabulous year of 2012 & 2013. The 2014 return is 2%, less than the effective withdrawal rate (8%*75% = 6%). This lead to the Non-Reserve fund shrunk in 2015. The reserve would be able to withstand about 10 years at the end of 2015 regardless of future recovery.

Parameter

To implement the reserve, we need many parameters.

- The initial reserve fund %
  - This parameter has major impact on the strategy. The purpose of this parameter is to set the reserve level. The reserve will be used in bad economic cycle and grown in good economic cycle. At the 25% reserve level, it reduces the II risk. See below II 95% Risk using ADD withdrawal rate of 6% at various E/B

Reserve	100%/0%	90%/10%	80%/20%	70%/30%	60%/40%	50%/50%
10%	24%	21%	20%	19%	20%	19%
15%	17%	16%	15%	11%	11%	12%
20%	11%	8%	6%	5%	4%	6%
25%	7%	5%	4%	4%	4%	4%
30%	8%	6%	5%	3%	3%	4%

  - Clearly we see that there's more than 3x more decrease in withdrawal at the 10% reserve than at the 25% level.
- The maximum reserve%
  - This parameter has a minor impact on the strategy. The purpose of this parameter is to rein in the reserve level when it gets too big. In prolong good economic cycle the reserve tends to deprive the member from substantial fund. Therefore once the reserve is above 35%, 50% of the excess above that is transfer to the regular fund for future use.
- The reserve protect protection
  - This parameter has a major impact on the strategy. This is the % of withdrawal we want to protect using the reserve. At the 95% protection level, we can think that 5% of withdrawal of any year is a bonus that may not be recurrent in case of bad year. Depending on the withdrawal rate, the bonus is very secure or unsecure. Using a 100% protection has an unpleasant surprise. If the economy is worse than projected, than the reduction in withdrawal quickly grows out of control. Ultimately this parameter should be driven by the person risk tolerance. All of the other recommended parameter "best fit" with the 95% level. If other level is selected, other set of "best fit" parameter should be used.
- The last parameter is the maximum withdrawal rate on reserve.
  - A maximum of 10% would imply that at the current rate, the reserve would last 10 more years. In the 10%-20% maximum withdrawal rate, there's not much difference in the various metric.

Data
The basic scenario is explained in more detail in the data section.
For this study, I've used both historical data and 500 Monte Carlo simulations.

Historical data

For the Smoothing
- I've used the information from 50 Years Data from http://www.fpsc.ca/news/publications-research/projection-assumption-guidelines - FPSC Projection Assumption Guidelines (4)

For the ADD model
- For Equity, use Robert Shiller data on S&P 500 as compiled by DQYDJ (5).
- For the Bonds, use Moody Seasoned AAA Corporate Bond Yield, (6)
- Inflation, from usinflationcalculator(7)
- 1908-1917 and 2016+ data,
    - Due to lacks of data on Bond before 1917, assumption are required prior 1917.
    - Assume 9.07% equity return.
    - Assume 4.29% bond return.
    - Assume 2% inflation

Monte Carlo data

For the Smoothing
- 10% returns to mimic returns on Equity
- 18% volatility to mimic volatility on Equity

For the ADD model
The Monte Carlo simulation of Equity based of the Projection assumption guidelines as of 2016-06-30 from Financial Planning Standards Council - IPQF(4). They have estimate to do projection using
- an inflation of 2.00%
- Fixed income (bonds) returns of 4.00%
- Canadian equities returns of 6.40%
- Foreign Developed market equities returns of 6.80%
- Emerging market equities returns of 7.70%

However, they have subtracted a safety margin on equities of 0.5% to reflect variability. Since the purposes are for a Monte Carlo, 0.5% will be added to the above equities return. Annual return on equities (33% Canadian/33% Foreign Equities (Developed)/33% Foreign Equities (Emerging)) as based IPQF historical data(4) is thus .

The US Federal bank have set a policy to have 2% inflation since 2012. (11) Bank of Canada have also set such limit in 1991. (12) FPSC 2% inflation projection seems appropriate. The inflation of Canada and US has been maintained since their announcement. Due to the timing of the change of policy of US, we looked at the post 1991 Canada inflation historical data to determine that the volatility was 0.72%.

While they acknowledge the use of Monte Carlo, they do not provide a guideline for the standard deviation to be use on Bonds or equities. To determine the standard deviation of 17.2632% on equities (33% Canadian/33% Foreign Equities (Developed)/33% Foreign Equities (Emerging)) as based IPQF historical data(4). The standard deviation for bonds instead was taken from FPSC historical bonds results of 7.74%. Next, we approximate the arithmetic average using $(1+G)^2 = (1+A)^2 - V$. Where G= Geometric Average, A= Arithmetic Average and V=Volatility(8). Correlation (0.15) is weak between the above portfolio (1/3 each equity) versus bonds. For the purpose of this study, we assumed that it was 0.

The starting Monte Carlo basis assumption are thus:

	Equity	Bonds	Inflation
Mean (Arithmetic)	7.47%	4.00%	2.00%
Volatility	17.2632%	7.744%	0.72%

One of the particularities of the bonds and equity is that they are not independent from year to year. They are not even log independent. We can demonstrate this by looking at the volatility for two years returns which are substantially lower than what the theory would predict for independent event. Instead of trying to come up with a fancy model to estimate the market, this study will create Monte Carlo simulation of 1 year return and combined 2, 4, 8, 16, 32 years returns. Each two data point is than adjusted to match the combined year return.

Ex:
Monte Carlo 1 year return: A1= 4%; A2=2%;
Monte Carlo 2 year return: B1= 15%
Adjusted A1= 8.2% = (1+4%)(((1+15%)/((1+4%)(1+2%)))^0.5)-1
Adjusted A2= 6.2% = (1+2%)(((1+15%)/((1+4%)(1+2%)))^0.5)-1
(1.082)(1.062)-1= 15%

This results in Monte Carlo to account for some mean reversion with many rebounding rate, while keeping the standard deviation, average and the randomness of the run intact.

For the inflation, there is no strong support for a mean reversion. Often the inflation remains high for years, or low for years. In addition, Canada release a statement that they have set an inflation control range of 1 to 3 percent. For our model thus we will base the inflation estimate on a normal curve. Results are limited to a lower bound of 1% and higher bound of 3%.

Conclusion

The study hopes is that it provides a useful strategy for all to better manage the risk in retirement. There's no reason to set a withdrawal rate that provides a risk of ruin. This strategy avoids it.

Another major hope is that accumulation phase and decumulation phase should be seen as a whole strategy and not as two separate phase of life. If you were able to have a risk tolerance at 100% Equity to Bond ratio, retirement should not change this tolerance. Retiree should not focus too much on the fund value but their withdrawal they can make. At the end of the day if you hold 10 shares of ABC, then you hold 10 shares of it until sold. If you want to get an estimate of the fund, it's better to look at the smoothed fund than the current fund value. You sale it at market price, but the market price does not force you to withdraw more or less. Therefore, the retiree should concentrate its attention to what he can withdraw not the fund worth. If the goal is to withdraw a small portion (less than 10% per year) the current market price is not what affects the most the strategy. The Market Value each time withdraw are made is what is important.

Furthermore, being 100% in the market, your fund would already ride the financial wave (up and down). By having kept in the market for a long time, you get substantial more benefit. The extra earnings also can be used to reduce the volatility risk.

A further area of study would be to look to remove the perpetual fund characteristic in order to provide even more stability in withdrawal.

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